Closing Tue, Jan. 12: 12.1, 12.2, 12.3
Closing Thu, Jan. 14: 12.4(1), 12.4(2)

# 12.4 The Cross Product

We define the cross product, or vector product, for two 3-dimensional vectors,

$\boldsymbol{a} = \langle a_1, a_2, a_3 \rangle$ and $\boldsymbol{b} = \langle b_1, b_2, b_3 \rangle$,

by $$\boldsymbol{a}\; x\; \boldsymbol{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} =$$

$$(a_2 b_3 - a_3 b_2)\boldsymbol{i} - (a_1 b_3 - a_3 b_1)\boldsymbol{j} + (a_1 b_2 - a_2 b_1)\boldsymbol{k}$$

*Ex*: If $\boldsymbol{a} = \langle 1,2,0 \rangle$ and $\boldsymbol{b} = \langle -1,3,2 \rangle$, then

$$\boldsymbol{a} \; x \; \boldsymbol{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 1 & 2 & 0 \\ -1 & 3 & 2 \end{vmatrix} =$$

$$( \qquad - \qquad )\boldsymbol{i} - ( \qquad - \qquad )\boldsymbol{j} + ( \qquad - \qquad )\boldsymbol{k}$$

## 12.5 Intro to Lines in 3D

In order to describe lines in 3D, we use parametric equations. Here is an example of 2D parametric lines:

*Ex*: Consider the 2D line: $y = 4x + 5$.

(a) Find a vector that is parallel to the line. Call this vector **v.**

(b) Find a vector whose head touches the line when drawn from the origin.
Call this vector $\mathbf{r_0}$.

(c) We can reach all other points on the line by walking along $\mathbf{r_0}$, then adding scale multiples of **v**.

This same idea works to describe any line in 2- or 3-dimensions.

**The equation for a line in 3D**:

Find

$\boldsymbol{v} = \langle a, b, c \rangle =$ a vector parallel to the line.

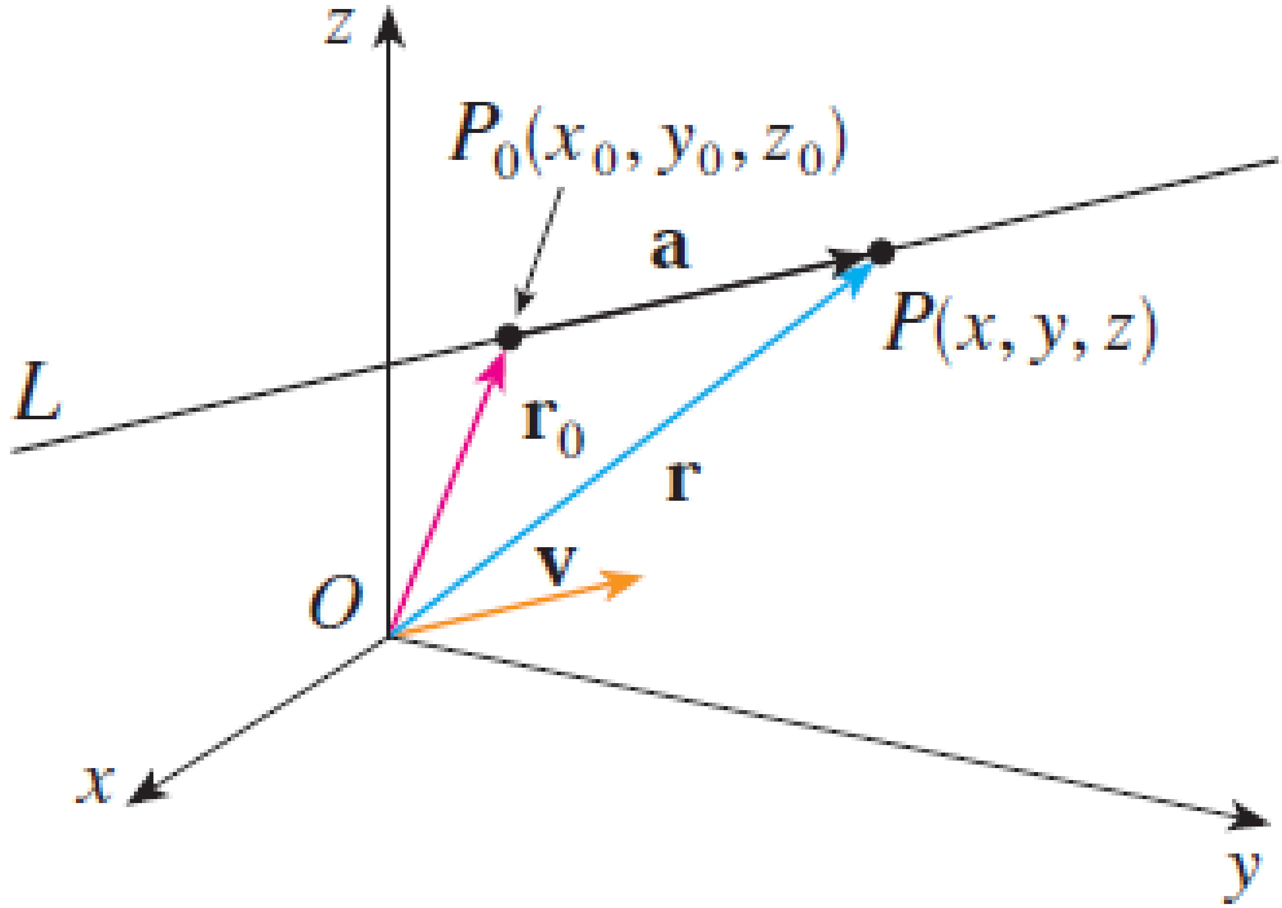
$\boldsymbol{r_0} = \langle x_0, y_0, z_0 \rangle =$ a position vector (*i.e.* $(x_0, y_0, z_0)$ is a point on the line)

Then all other points on the line can be obtained by

$$\langle x, y, z \rangle = \langle x_0, y_0, z_0 \rangle + \mathrm{t}\langle a, b, c \rangle,$$

which is sometimes written as:

$$\boldsymbol{r} = \boldsymbol{r_0} + \mathrm{t}\,\boldsymbol{v}$$


z
$P_0(x_0, y_0, z_0)$
a
$P(x, y, z)$
L
$\mathbf{r}_0$
r
v
O
x
y